Lecture 14

Friday, February 12, 2021 4:15 PM

* Prayer
* Spiritual thought
* Answering questions

Partial derivative:

$$f(x,y) = x^2 + xy - y^3$$

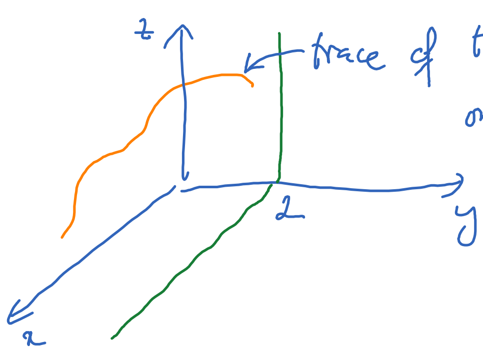
$$\rightarrow \quad f_x(x,y) = 2x + y$$

what about $f_x(1,2)$?

Fix $y=2$, we get a function $g(x) = f(x,2)$.

Then take the derivative of f at $x=1$.

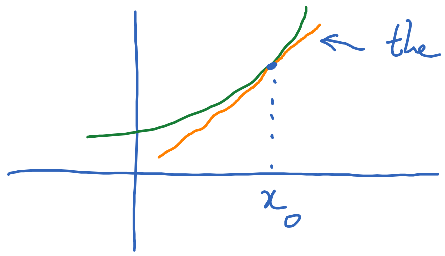
$$f_x(1,2) = g'(1).$$



trace of the graph of f on the plane $y=2$.

Linear approximation

Recall: $f(x)$



← the line that best approximates f near x_0.

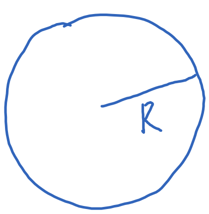
That line (the tangent) passes through $(x_0, f(x_0))$ and direction vector $\langle 1, f'(x_0)\rangle$.

$$\begin{cases} x = x_0 + t \\ y = f(x_0) + f'(x_0)\, t \end{cases} \rightsquigarrow y = f(x_0) + f'(x_0)(x - x_0).$$

What is the purpose of this approximation?

- Linearizing the problem.
- Approximate the error. $\Delta f \approx f'(x) \Delta x$

Ex



Use machine to make a pizza of radius R. If there is an error in R, say $\varepsilon(R)$, what is the error in the area of the pizza?

$$S = \pi R^2$$

$$\Delta S \approx 2\pi R \Delta R \lesssim 2\pi R \varepsilon.$$

Now consider $f(x,y)$

$(x_0, y_0, f(x_0,y_0))$

z_0

$$f(x_0+h, y_0+k)$$

$$\approx f(x_0, y_0+k) + h f_x(x_0, y_0+k)$$

$$\approx f_y(x_0,y_0)k + f(x_0,y_0) + $$

$$+ h\left(f_x(x_0,y_0) + k f_{xy}(x_0,y_0)\right)$$

$$\approx z_0 + f_x(x_0,y_0)h + f_y(x_0,y_0)k.$$

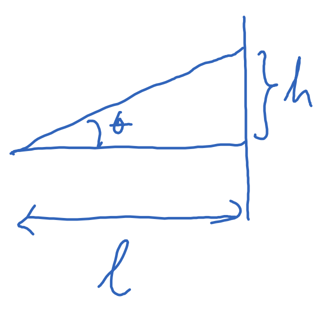
Tangent plane: $z = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0) + z_0$

Ex: Find the equation of the tangent plane to the ellipsoid

$$2x^2 + 2y^2 + z^2 = 2$$

at point $\left(\frac{1}{2}, \frac{1}{2}, 1\right)$.

Ex:



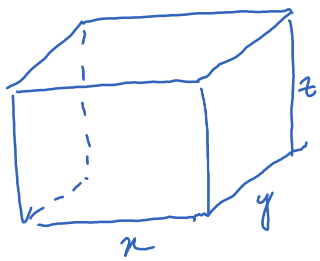
Shooting machine:

$$h = h(\theta, l) = l\tan\theta.$$

If $\theta = \frac{\pi}{4} + \varepsilon_1$ and $l = 20 + \varepsilon_2$

then what is Δh approximately?

Ex:



$V = xyz$

What is ΔV (approximately)?